

Possible Answers:

$$\begin{aligned} 1.) \quad \sin 4\theta &= 2 \sin 2\theta \cos 2\theta \\ &= 2(2 \sin \theta \cos \theta)(1 - 2 \sin^2 \theta) \quad \text{OR} \\ &= 4 \sin \theta \cos \theta (1 - 2 \sin^2 \theta) \end{aligned}$$

$$\begin{aligned} 1.) \quad \sin 4\theta &= 2 \sin 2\theta \cos 2\theta \\ &= 2(2 \sin \theta \cos \theta)(\cos^2 \theta - \sin^2 \theta) \\ &= 4 \sin \theta \cos \theta (\cos^2 \theta - \sin^2 \theta) \end{aligned}$$

$$\begin{aligned} 2.) \quad \cos 8\theta &= \cos^2 4\theta - \sin^2 4\theta \\ &= (\cos 4\theta)^2 - (\sin 4\theta)^2 \\ &= (\cos^2 2\theta - \sin^2 2\theta)^2 - (2 \sin 2\theta \cos 2\theta)^2 \\ &= \left((\cos 2\theta)^2 - (\sin 2\theta)^2 \right)^2 - \underbrace{\left(2(2 \sin \theta \cos \theta) \cdot (\cos^2 \theta - \sin^2 \theta) \right)^2}_{\text{done}} \\ &= \left((\cos^2 \theta - \sin^2 \theta)^2 - (2 \sin \theta \cos \theta)^2 \right)^2 - \left(2(2 \sin \theta \cos \theta) \cdot (\cos^2 \theta - \sin^2 \theta) \right)^2 \end{aligned}$$

$$\begin{aligned} 3.) \quad \sin 3\theta &= \sin(2\theta + \theta) \\ &= \sin 2\theta \cos \theta + \cos 2\theta \sin \theta \\ &= 2 \sin \theta \cos \theta \cos \theta + (1 - 2 \sin^2 \theta) \sin \theta \\ &= 2 \sin \theta \cos^2 \theta + \sin \theta - 2 \sin^3 \theta \\ &= 2 \sin \theta (1 - \sin^2 \theta) + \sin \theta - 2 \sin^3 \theta \\ &= 2 \sin \theta - 2 \sin^3 \theta + \sin \theta - 2 \sin^3 \theta \\ &= 3 \sin \theta - 4 \sin^3 \theta \end{aligned}$$